A NEW TOOL FOR HARMONIC ANALYSIS: VALENCE THEORY
(musical examples all at end)

While music theory has attained great penetration in the study of linear activity, long-range form, and overall tonal directedness, there has been a relative lack of thorough investigation of the expressiveness and subtleties of foreground harmonic events. Composers such as Mussorgsky, Gesualdo and Debussy frequently make their deepest comments along this line, and many more composers, especially in the last 100 years, can be studied, at least partially, from this viewpoint.

In order to understand such composition better, it is desirable to use a numerical scheme to make precise the degrees of harmonic closeness or distance, either within a chord or between consecutive chords. The acoustical basis is the axiom that notes related to one another by perfect 5ths are maximally close harmonically. According to this axiom, the term <u>valence</u>, borrowed from chemistry, is applied to pitches according to the following table:

Fb	Cb	Gb	Db	Ab	Eb	Bb	F	C	G	D	A	E	B	F#	C#	G#	D#	A#	E#	B#
-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	+1	+2	+3	+4	+5	+6	+7	+8	+9	+10

The choice of zero for D is for numerical convenience and symmetry. As all conclusions are based on relationships between valences, any note could have been selected as the "zero."

It will be observed that enharmonic spellings of the same "piano" pitch have considerably different valences, and therefore that the spelling of passages to be analyzed with this tool must be carefully considered.

Generally speaking, two notes of nearly equal valences will be present, diatonically, in a relatively large vocabulary of possible modes or tonalities, while notes of distant valences will tend to be related only chromatically.

For a chord, or any other collection of pitches (such as the entire set of notes covered by a given melody, or all the notes of a particular scale) one can compute the overall valence of the collection as the simple average of the valences of the notes. This may be made mathematically rigorous[1] thusly: Let K be a chord (or collection) of n tones (t_1, $t_2 \dots t_n$) having the valences v_1, $v_2 \dots v_n$. Then the valence of K (V_K) is given by the formula:

$$V_K = \frac{\sum_{i=1}^{n} v_i}{n}$$

The valence of a chord measures its relative position as an aggregate of notes along a continuum from "the flat side" to "the sharp side." This is but one property of a chord and admittedly does not take into account the inversion, doubling, spacing or surrounding voice-leading.

The valence of a C Major chord, for example, is $\frac{-1}{3}$; G Major is $\frac{+2}{3}$. In fact any major triad will have a valence of the form $\frac{3x+2}{3}$, or more precisely $\frac{3v_r+5}{3}$ where v_r is the valence of the root as a separate pitch. All minor triads will have valences of the form $\frac{3x+1}{3}$, or $\frac{3v_r-2}{3}$. This numerical idiosyncracy is unimportant in itself but it does have some interesting corollaries; firstly, no individual pitch (with whole integer valence, of course) has identical valence to any major or minor triad; secondly, no two major or minor triads have precisely equal valences. Therefore, any progression between consonant chords involves some "motion" in the sense of valence-change or sharpness-flatness. If one removes the consonance restriction, there are indeed progressions of zero valence change, some of which will be detailed later.

Given two chords or collections K_1 and K_2, we clarify the concept of valence change as follows. The remoteness (R) of the progression from K_1 to K_2 is defined as the difference between their valences:

$$R_{(K_1,K_2)} = V_{K_2} - V_{K_1}$$

When $R_{(K_1,K_2)}$ is a positive number the progression is towards the "sharp side"; when a negative number, towards the "flat side." We will give the remoteness as a positive number when we are discussing two chords or collections without regard to a direct consecutive connection.

We can observe that most connections in diatonic situations have relatively low remoteness, specifically less than 2.

Ex. 1

In particular, the standard tonal ending V_7-I is of especially low remoteness, most notably in the case where the V_7 is incomplete.

Ex. 2

The viability of the classical incomplete dominant, with missing 5th, is in part due to the fact that the excised note makes nearly zero difference in the computation of the valence of the entire chord; stated otherwise, the excised note has a valence nearly equal to that of the chord as a whole:

Ex. 3

The ensuing connections are considerably more chromatic.

Ex. 4 Puccini-Tosca

Ex. 5 Strauss-Elektra

In both passages we note relatively high remoteness, but in Ex. 4 the overall connection, $K_1 \longrightarrow K_3$, is maximally remote; the middle chord (K_2) is more or less transitional. In Ex. 5 the overall connection $K_1 \longrightarrow K_3$, is minimally remote,- in fact quite diatonic, but the middle chord (K_2) provides great remoteness from either of them, despite the powerful common tone. Ex. 4 has a feeling of accumulating chromaticism, where Ex. 5 has more of an arch-structure of chromatic tension. Note that the concept of remoteness, while essentially a property of pairs of chords, can be of analytical value in progressions of more than two chords.

The connection of two chords of the same quality with roots a tritone apart will be seen to have remoteness equal to 6. This would appear to be a maximum, as connections of greater remoteness could be respelled to give lower remoteness, but such respelling may not always be true to the intervallic nature of the music. Exs. 6 and 7 employ the same relationship, with remoteness of $\frac{19}{3}$. In both cases the spectacular connection opens the work, to great theatrical effect.

Ex. 6 Gesualdo-*Moro Lasso*

Ex. 7 Vaughan Williams-*A Sea Symphony*

Ex. 6 expresses great despair while Ex. 7 expresses adventure and optimism, at least based on the texts. In Ex. 6 the progression is from a major chord to a minor chord and "to the flat side" (negative remoteness); the opposite situation prevails in Ex. 7. The question of the poetic effect of progressions of positive or negative remoteness may or not be meaningful, although one can note that most firm cadences in traditional music feature negative remoteness, with the exception of the so-called Phrygian cadence, which is relatively unsatisfying, and the Plagal cadence, which is often not a true cadence at all, but a simple "neighbor-chord" effect subsequent to a dominant cadence.

The valence concept may also be used to study the internal complexity of a chord, with respect to self-contained chromatic tendancies. This is naturally correlated with the nearness or distance (in terms of valence) of the individual notes from the chord as a whole. We will define the <u>eccentricity</u> (E) of a chord (K) as the average difference in valence between the notes of K and the valence of the whole chord. Note that we consider the differences as absolute values, not considering the positive or negative sign; this allows the <u>eccentricity</u> to represent accurately the <u>accumulated</u> chromatic involvement of the sound.

There is some question as to which of two concepts should be more useful; one is the average of the distances from each note to the whole chord, and the other is the total of those distances. In the first case the addition of a new note of mild chromatic effect can soften the overall eccentricity, while in the second case any new note is considered to thicken the result. For example, using average eccentricity, the major 7th chord is less eccentric than the two-note sonority of its outer notes, but using total eccentricity the chord is more eccentric than the interval. For the remainder of this paper, we are using average eccentricity.

The formula is:

$$E_K = \frac{\sum_{i=1}^{n} |V_K - v_i|}{n}$$

where the v_i are the valences of the n notes of K and where V_K is the valence of the chord. Again, note that we are adding the absolute values ($| \text{--} |$). (If we were to compute the total eccentricity of K we simply would not divide by n.)

As both the Valence and Eccentricity of the chord involve averaging, we can expect some relatively clumsy fractional numbers. For example, the eccentricity of any major and minor triad is precisely $\frac{14}{9}$. Eccentricity is not exactly a measure of dissonance, though it has some correlation with it. The main area of non-correlation involves the fact that thirds are more eccentric than major seconds. Therefore the chord built in fourths or fifths (e.g. Bb-F-C) has considerably lower eccentricity ($\frac{2}{3}$) than a consonant triad.

An analysis that combines study of remoteness and eccentricity can often be quite revealing. For example in the traditional ending shown in Ex. 8, we observe the progression It_6-V-I

Ex. 8

The augmented 6th chord has high eccentricity but its valence is relatively conservative ~~and~~ in that there is low remoteness in the entire progression. Moreover, the final chord has, as its valence, the average of the valences of the first two chords. Such a valence may be a factor in helping chords to sound final or cadential. Finally, note that the Italian chord has exactly the same valence as the IV of the same key, which is its traditional functional co-equal.

Among other applications these theories may lead to are those pertaining to linear events in a harmonic context. So called "non-harmonic" tones, while rarely contributing to entire new chords in their linear existence, most certainly lend harmonic dimension to the passages in which they occur according to their relative sharpness or flatness to the prevailing activity.

Let us consider two chords with one linearly moving note. We can examine three valences,- V_{K_1}, V_{K_2} and v_t, where K_1 and K_2 are the chords and t is the moving note. It is not important whether $R_{(K_1,K_2)}$ is positive or negative but we can observe whether the valence of t is in one of three positions, namely

1. v_t-----V_{K_1}-----V_{K_2} ,

or 2. V_{K_1}----v_t------V_{K_2} ,

or 3. V_{K_1}-----V_{K_2}----v_t.

where the dotted lines represent either all <u>less than</u> signs or all <u>more than</u> signs ($<$, $>$) depending on which chord has higher valence. Therefore, situation 2. represents the case where the valence of the linear note is between the two chord valences, situation 1. represents the case where the valence of the linear note is not between the two chordal valences and is closer to the valence of K_1, and situation 3. represents the case where it is closer to K_2.

From initial inspection of examples, these three possible cases[2] lead to very definite profiles in terms of emotional effect. For this reason, we term linear notes as in situation 1. <u>backward intensifying</u>, those as in situation 2, <u>soft</u>, and those as in situation 3. <u>forward intensifying</u>.

Observe, in the standard Landino-Bergundian ending, the extreme chordal remoteness.

Ex. 9

The effect is softened by the note t (the Landino note), despite the fact that it is left by skip.

The diatonic progression of Ex. 10 is intensified by the linear note, in a way that creates a feeling of longing and prolongation of the first chord, while the second sounds dark and somewhat yearning. Note the change in Ex. 11, where the progression is still intensified but now the last chord sounds more final and resigned. Ex. 12 is still a third version with soft linear note.

Ex. 10 Rochberg- Violin Concerto

Ex. 11

Ex. 12

Ex. 13 provides an instance of a strong forward-intensifying note. The actual progression from K_1 to K_2 is somewhat notable in the change in eccentricity, with moderate remoteness[3]. But the linear note gives great added power to the succession, providing its harmonic effect despite its short duration and attack on an extremely weak pulse.

Ex. 13 Rosner-Five Ko-ans for Orchestra

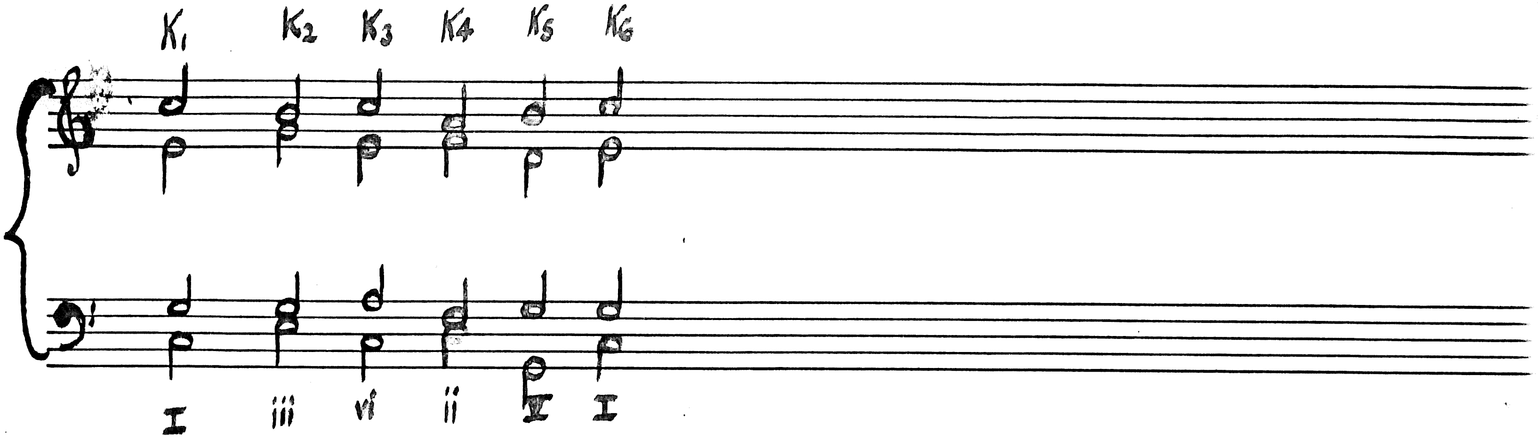
In closing, it should again be pointed out that these ideas are in no way intended to replace any other shhemes of musical analysis. I am proposing but one new device to add to the arsenal of music theory, which may prove useful particularly with music that employs harmony and harmonic implications for much of its communicative power. Even in such works, no thorough understanding would be obtained from these theories unless they were well integrated into a more complete analytical scheme.

Arnold Rosner

1. The mathematical formulas are shown to simplify the English explanations for those who prefer abstract equations to their verbal description. In no case, however, have I omitted a full textual definition so those who do not prefer formulas may simply ignore them.
2. There are some other possible cases but these would be very infrequent and may not contribute anything new to the discussion. v_t may be exactly equal to the valence of one of the chords. Or v_t may not lie between the chord valences but may be equally close to either one according to enharmonic spelling.
3. If Eb were used instead of D#, the C would be backward intensifying. As the triad is augmented in any case, and in neither spelling in root position, it seems reasonable to spell according to the linear movement, which justifies D#. Moreover, if our theories have any validity, the driving rather than prolonging effect of the C would tend to imply the correctness of the D# spelling. This last point is, of

course, hardly conclusive as it constitutes circular reasoning at this stage, but it is offered as a provocative suggestion of where such investigation may lead.

Ex. 1

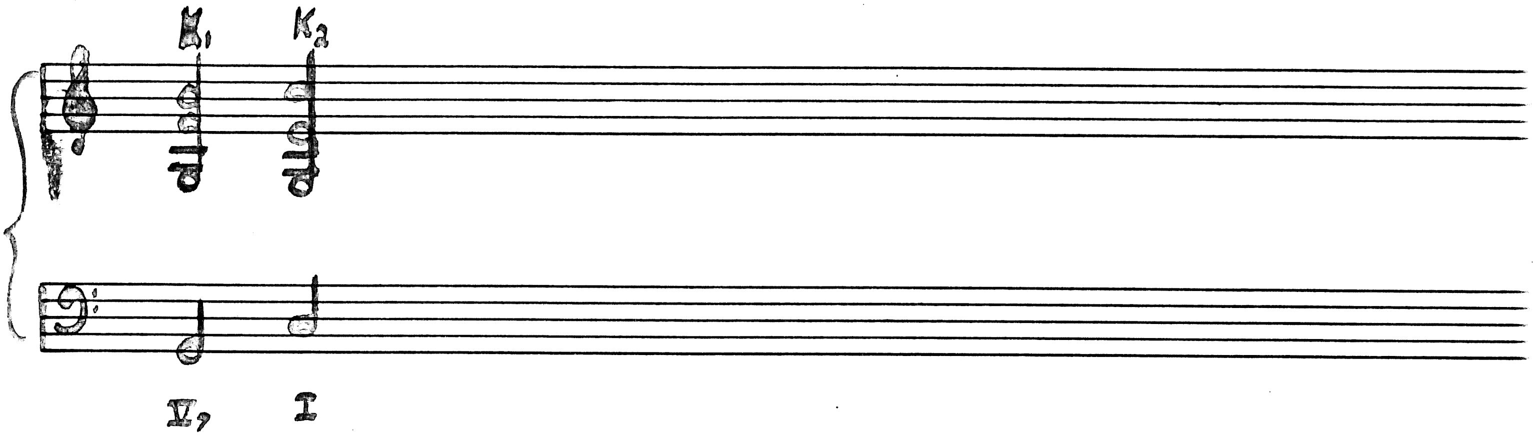


$V_{K_1} = \frac{-1}{3}$ $V_{K_2} = \frac{+4}{3}$ $V_{K_3} = \frac{+1}{3}$ $V_{K_4} = \frac{-2}{3}$ $V_{K_5} = \frac{+2}{3}$ $V_{K_6} = \frac{-1}{3}$

LR = $+\frac{5}{3}$ LR = -1 LR = -1

LR = -1 LR = $\frac{+4}{3}$

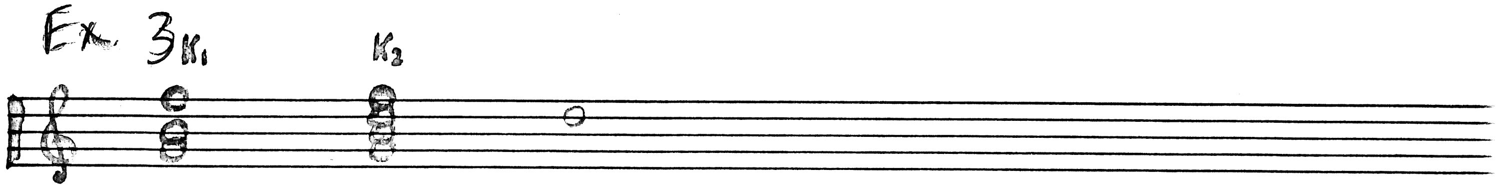
Ex. 2



$V_{K_1} = \frac{-1}{3}$ $V_K = \frac{-1}{3}$

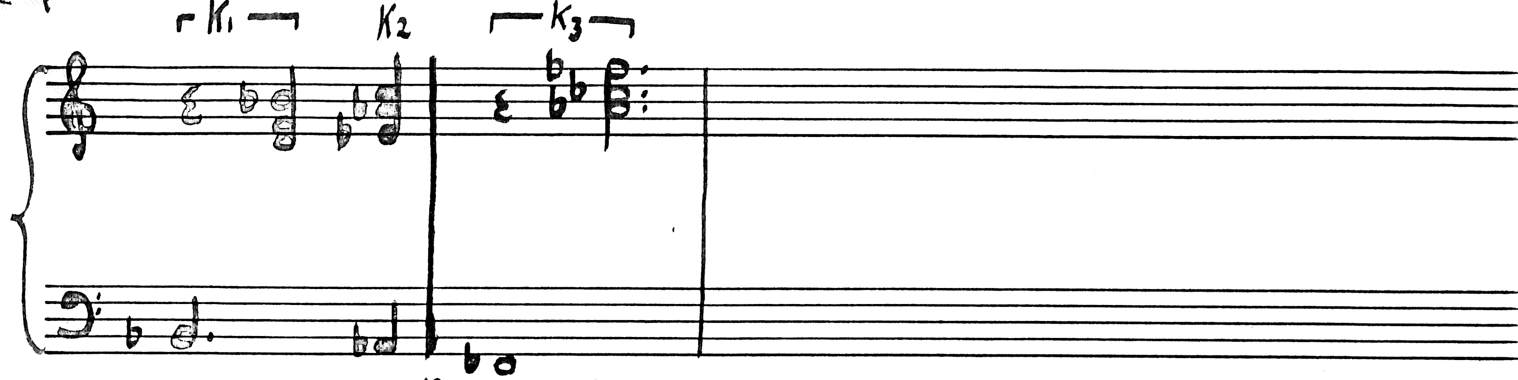
LR = 0

Ex. 3



$V_{K_1} = \frac{-1}{3}$ $V_{K_2} = \frac{-1}{4}$ $r = 0$

Ex. 4

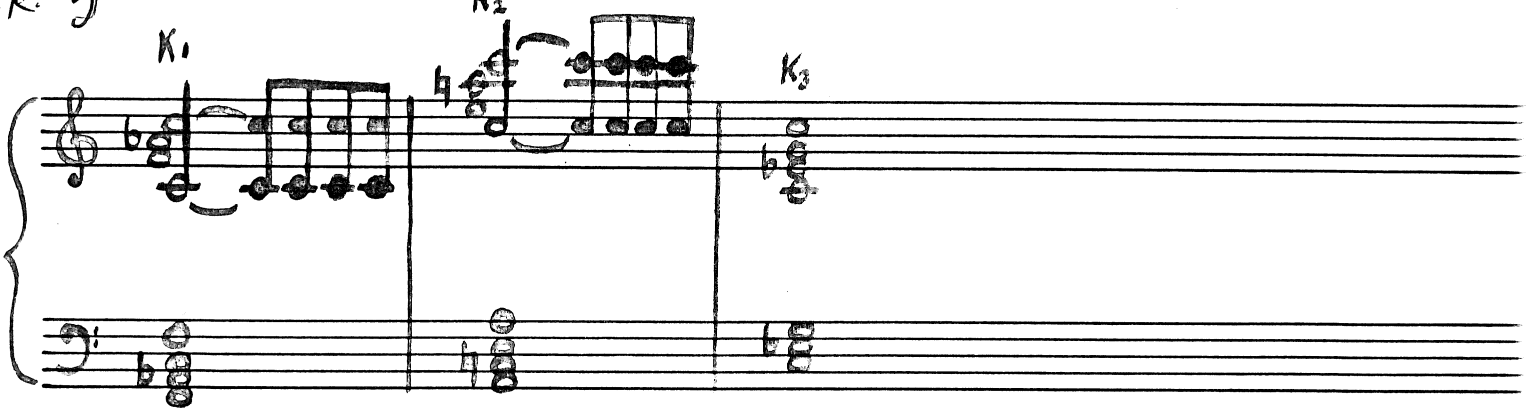


$V_{K_1} = \frac{-7}{3}$ $V_{K_2} = \frac{-13}{3}$ $V_{K_3} = \frac{-25}{3}$

$R = -2$ $R = -4$

$R = -6$

Ex. 5

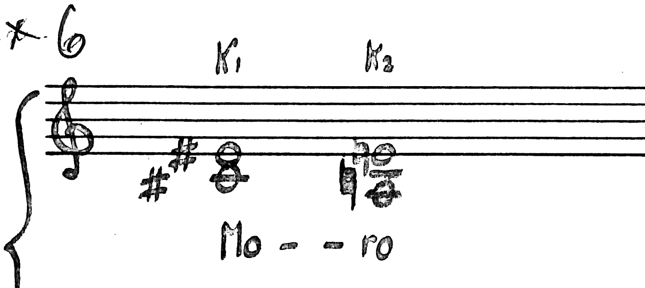


$V_{K_1} = \frac{-11}{3}$ $V_{K_2} = \frac{+1}{3}$ $V_{K_3} = \frac{-8}{3}$

$R = +4$ $R = -3$

$R = +1$

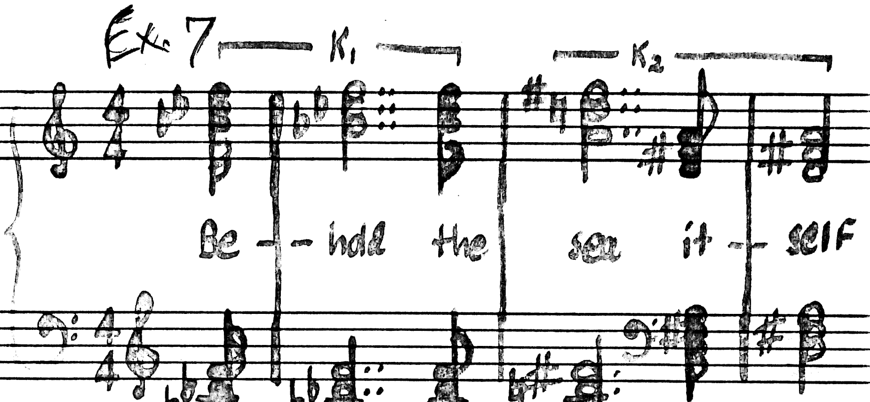
Ex. 6



$V_{K_1} = \frac{+20}{3}$ $V_{K_2} = \frac{+1}{3}$

$R = \frac{-19}{3}$

Ex. 7



$V_{K_1} = \frac{-14}{3}$ $V_{K_2} = +\frac{5}{3}$

$R = +\frac{19}{3}$


Ex. 8
E = 32/9 E = 14/9 E = 14/9
K1 K2 K3
VK1 = 4/3 VK2 = +2/3 VK3 = -1/3
R = +2 R = -1
Ex. 9
K1 t K2
VK1 = +13/3 VK2 = +1/2
Vt = 3
VK1 < Vt < VK2
Ex 10
K1 t K2
VK1 = +1/3 Vt = 3 VK2 = -4/3
R = -5/3
Vt > VK1 > VK2
Ex 11
K1 t K2
VK1 = +1/3 Vt = -4 VK2 = -4/3
VK1 > VK2 > Vt
Ex 12
K1 t K2
VK1 = +1/3 Vt = -1 VK2 = -4/3
VK1 > Vt > VK3
Ex 13
E = 8/3
K1 t
E = 1/2
K2
VK1 = +3 Vt = -2
R = -3/2
VK1 > VK2 > Vt